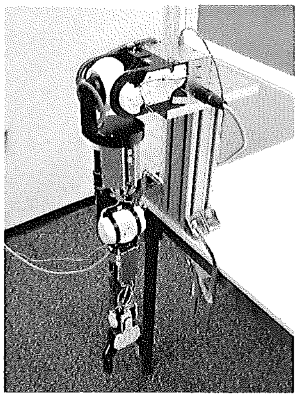
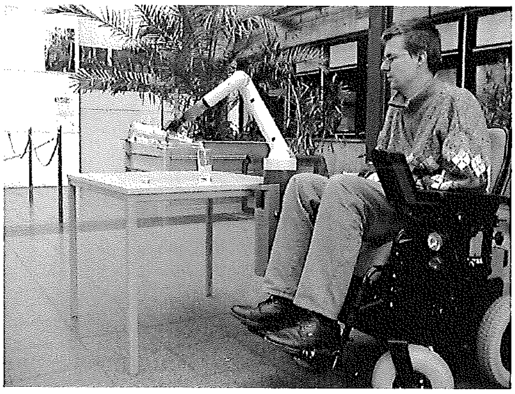
Exam Mechatronics, 31 January 2014

Name: Student ID:

Exercise 1 (30 points)

Consider the Philips Experimental Robotic Arm as can be found in the lab of DTPA. The arm resembles a humanoid arm, is able to measure position and forces at the joints, and has motors in the joints, resulting in a 7 DOF (Degrees of Freedom, directions of movement) robotic arm to move like a humanoid arm. Philips has developed the arm for R&D purposes, since they want to innovate their domestic applications, i.e., the arm should help with tasks that are currently done by humans in the house, or to be used for disabled people, where the arm is attached on a wheel chair, such as in the picture below. The control of the arm is ideally done by micro-controllers.





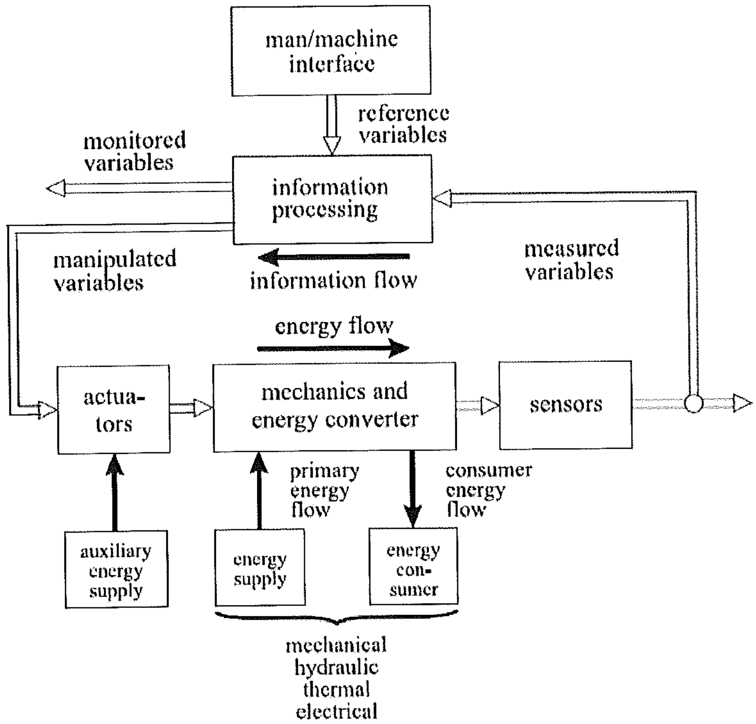
a). Identify two possible user demands, two possible functional requirements (FR) and two possible design parameters (DP). **(8 points)**

User demands: - pick up things safely
~~- respond properly to special tasks~~
- easy to use
- safe to use
- ...

FR: - batteries smaller than x m^3, with a power supply of at least y Watts, and weight less than z kg.
- overall weight less than a kg.
- lifetime of the arm $>$ b years
- accuracy of picking up things high enough
- ...

DP. - choice for motors
- sensors for accurate control
- design of all elements to meet life time requirements
- ...

b). Consider the mechatronic diagram (Iserman 2008)



Identify the actuators, sensors, and information processing. Also provide an idea for a possible man/machine interface. **(7 points)**

actuators: motors in joints

sensors: position and force sensors

information processing: micro controllers
~~he~~ controlling the actuators.

man/machine interface: - a box with a small user interface, where it can be specified what the task is..
- or speech recognition, ~~i.e.~~ so that arm reacts at a spoken command.

c). The gripper dynamics are described by

$$\begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{m} \\ -K & -\frac{d}{m} \end{pmatrix} \begin{pmatrix} q \\ p \end{pmatrix} + \begin{pmatrix} 0 \\ B \end{pmatrix} u$$

where m is the mass, K a spring constant as present in the gripper, and d the damping. q, and p are the position and momentum, respectively. u is the actuator force. Take $m = K = d = B = 1$, and as output the position $-q$. Design a PD controller such that the eigenvalues (or poles) of the closed loop system are given by -2 and -3. **(7 points)**

$$\begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}}_{A} \begin{pmatrix} q \\ p \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{B} u$$

$$y = \begin{pmatrix} -1 & 0 \end{pmatrix} \begin{pmatrix} q \\ p \end{pmatrix}$$

$$H(s) = C(sI - A)^{-1} B$$

$$\begin{pmatrix} s & -1 \\ 1 & s+1 \end{pmatrix}^{-1} = \frac{1}{s^2+s+1} \begin{pmatrix} s+1 & 1 \\ -1 & s \end{pmatrix}$$

$$H(s) = \frac{-1}{s^2+s+1}$$

$1 + C(s)H(s) = 0$ determines the closed loop poles.

$$1 - \frac{K_p + K_d s}{s^2+s+1} = 0 \iff s^2+s+1-K_p-K_d s = 0$$

Desired char. pol. : $(s+2)(s+3) = s^2+5s+6$

$\iff K_d = -4$ and $K_p = -5$

(normally $K_p, K_d > 0$, but due to the strange choice of y, they now become negative to make it work, i.e., $y = -q$ counteracts the $-$ sign in the loop).

Hence $C(s) = -5 - 4s$

d). Find the implementation of the PD controller from exercise c) by a discrete-time controller $C(z)$ using the bilinear approximation with $T = 2$, and determine the corresponding difference equation! **(8 points)**.

$-5-4s$

$-5-\frac{4-4z^{-1}}{1+z^{-1}}$ suppose e in u out

$\frac{U(z)}{E(z)} = \frac{-5-5z^{-1}-4+4z^{-1}}{1+z^{-1}}$

$E(z)(-9-z^{-1}) = U(z)(1+z^{-1})$

$-9e(k)-e(k-1) = u(k)+u(k-1)$

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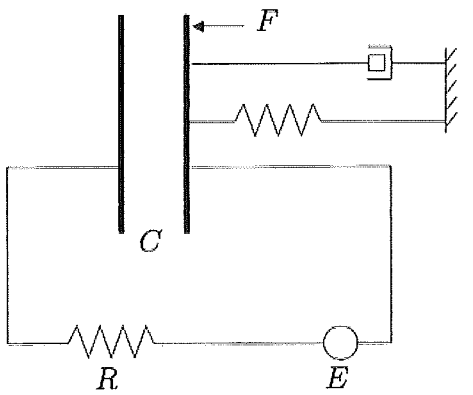
Exercise 2 (20 points)

Note that the Euler-Lagrange equations are given by

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}(q,\dot{q})\right) - \frac{\partial \mathcal{L}}{\partial q}(q,\dot{q}) = -\frac{\partial \mathcal{D}}{\partial \dot{q}} + Gu$$

where $\mathcal{L}$ is the Lagrangian, $\mathcal{D}$ the Rayleigh dissipation function, and G the input matrix.
Consider a capacitor microphone system as follows



x: linear displacement of the capacitor plates
$C(x)$: capacitance.
κ: dielectric constant
A: surface of the plate
R: resistor constant
E: voltage source
B: damping coefficient
m: mass of the plates of the capacitor
k: spring constant

where the capacitance is determined from measurements, and is given by $C(x) = \frac{\kappa A}{x}$, $x \neq 0$, and depends on the mechanical displacement x. The coupling between the mechanical and electrical domain is thus going via the capacitance $C(x)$. Applying a force at the plate (resulting from the sound) results in a change of distance between the plates of the capacitor. The voltage source ensures that the charge on the plates is constant, and hence, the distance changes cause a change in voltage over the capacitor, resulting in a signal that can be used. The spring, mass (can be modeled as only one mass), and the damper and resistor are all linear. Note that the electric energy of the capacitor, $\frac{1}{2C(x)}q_e^2$ can be seen as potential energy, with q_e the charge on the capacitor.
The coupling force at the mechanical side is given by

$$\frac{1}{2\kappa A}q_e^2$$

and the coupling voltage at the electrical side is given by

$$\frac{x}{\kappa A}q_e$$

a). Provide the Euler-Lagrange equations of the above system. Motivate your answer! **(15 points)** Note: if this is a problem, you may consider to determine the equations of motion according to the standard conservation laws. **(in that case 7 points)**.

$$\mathcal{L} = \frac{1}{2} m\dot{x}^2 - \frac{1}{2} K x^2 - \frac{x}{2\kappa A} q_e^2$$ 5

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} = \begin{pmatrix} m\ddot{x} \\ 0 \end{pmatrix}$$

$$\frac{\partial \mathcal{L}}{\partial q} = \begin{pmatrix} -Kx - \frac{1}{2\kappa A} q_e^2 \\ -\frac{x}{\kappa A} q_e \end{pmatrix}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}(q,\dot{q})\right) - \frac{\partial \mathcal{L}}{\partial q}(q,\dot{q}) = \begin{pmatrix} m\ddot{x} + Kx + \frac{1}{2\kappa A} q_e^2 \\ \frac{x}{\kappa A} q_e \end{pmatrix}$$ 4

$$-\frac{\partial D}{\partial \dot{q}} + Gu = \begin{pmatrix} F - B\dot{x} \\ E - R\dot{q}_e \end{pmatrix} \begin{matrix} \rightarrow 3 \\ \rightarrow 3 \end{matrix}$$

b). How many states are minimally needed to describe the above system provided that we are interested in the position x? Motivate your answer! **(5 points)**.

3 states, including x, $\dot{x}$, and q_e, as $\dot{q}_e$ is not present and it is a 2^{nd} order equation.

This can be seen from the energy storing elements,
i.e. a spring (state x)
a mass (state $\dot{x}$), and
a capacitor (state q_e)

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Exercise 3 (30 points + 7 bonus)

The first part on this exercise concerns optimal state feedback design (LQR design), and the second part concerns state observer design.

a). If we find one positive-definite matrix P as a solution to the Riccati equation, what implication does this have for the system? (**6 points**)

There is a feedback that makes the closed loop system stable.

See note on Nestor week 6

b). Consider the system described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u, \qquad (1)$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Determine the optimal feedback gain matrix K such that the following performance index is minimized:

$$J = \int_0^\infty x_1^2(\tau) + 4x_2^2(\tau) + 4x_1(\tau)x_2(\tau) + 9u(\tau)d\tau$$

(8 points)

$$\begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} - \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\frac{1}{9}(1\ 0)P + \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow 2P_{21} + 2P_{12} - \frac{1}{9}P_{11}^2 + 1 = 0$$

$$\left.\begin{aligned} 2P_{22} - \frac{1}{9}P_{11}P_{12} + 2 = 0 \\ 2P_{22} - \frac{1}{9}P_{11}P_{21} + 2 = 0 \end{aligned}\right\} P_{12} = P_{21}$$

$$-\frac{1}{9}P_{21}P_{12} + 4 = 0$$

Solve and find

$$P = \begin{bmatrix} 15 & 6 \\ 6 & 4 \end{bmatrix} \Rightarrow K = \begin{bmatrix} \frac{5}{3} & \frac{2}{3} \end{bmatrix}$$

c). For the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u,$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{2}$$

let a state observer of the following form be designed:

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u, +L(y - \hat{y})$$
$$\hat{y} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}. \tag{3}$$

where $\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix}$ is the estimated state, $\hat{y}$ is the estimated output and L is the observer gain. Define $\tilde{x} = x - \hat{x}$ to be the estimation error. Write down the characteristic polynomial of the error system corresponding to an estimate of x_2 with a (exponential) convergence rate of 10 (i.e. $||x(t) - \hat{x}(t)|| = ||x(0) - \hat{x}(0)||exp(-10t)$). Motivate your answer. (**8 points**)

See tutorial on observer design

$(\lambda + 10)(\lambda + 10) = \lambda^2 + 20\lambda + 100$

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d). Write down the observer gain $L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$ corresponding to the convergence rate in **(c)**, based on your answer in **(c)**. (In case you did not find a solution to the previous exercise, assume that the characteristic polynomial of the error system is $\lambda^2 + 5\lambda + 20 = 0$ and proceed with the calculations to find L.) (**8 points**)

$$\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} - \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}$$

$$= \begin{bmatrix} -l_1 & 1 \\ 2-l_2 & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}$$

$$\det\left(\lambda I - \begin{bmatrix} -l_1 & 1 \\ 2-l_2 & 0 \end{bmatrix}\right) = \lambda^2 + \lambda \underbrace{l_1}_{} \underbrace{-2+l_2}_{}$$
$$= \lambda^2 + 20\lambda + 100$$

$$\Rightarrow \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 102 \end{bmatrix}$$

e). Bonus question. Explain how the separation principle works. Why can we separate the controller and observer design? **(7 points)**.

Starting: $\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$ Controller design to steer system to x^* corresponding to u^*

Define $\tilde{x} = x - x^*$, $\tilde{u} = u - u^*$

$\tilde{u} = -BK\tilde{x}$ then $\dot{\tilde{x}} = A\tilde{x} - BK\tilde{x}$

Estimator & $\hat{x}$

$$\dot{\hat{x}} = A\hat{x} + L(y - \hat{y})$$
$$\hat{y} = C\hat{x}$$

Error $e_x = x - \hat{x}$, $e_y = y - \hat{y}$

$$\begin{cases} \dot{e}_x = (A - LC)e_x \\ e_y = Ce \end{cases}$$

Now take $\underline{\tilde{u} = -BK(\hat{x} - x^*)}$ i.s.o. $\tilde{u} = -BK(x - x^*)$

then $\tilde{u} = -BK(x - e_x - x^*) = -BK(\tilde{x} - e_x)$

then $\begin{cases} \dot{\tilde{x}} = (A - BK)\tilde{x} + BKe_x \\ \dot{e}_x = (A - LC)e_x \end{cases}$

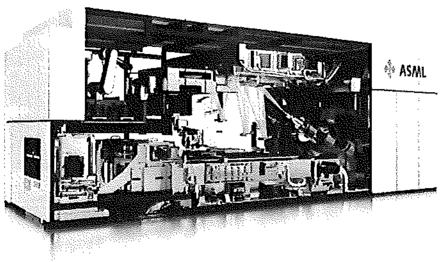
Overall $\bar{A}$ matrix: triangular structure.

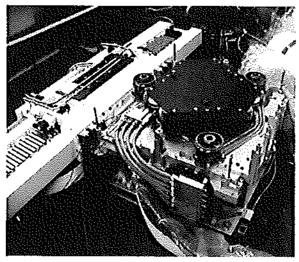
$$\begin{pmatrix} \dot{\tilde{x}} \\ \dot{e}_x \end{pmatrix} = \begin{pmatrix} A - BK & BK \\ 0 & A - LC \end{pmatrix} \begin{pmatrix} \tilde{x} \\ e_x \end{pmatrix}$$

Stability corresponds to eigenvalues of $A - BK$ and $A - LC$ in LHP. Hence, design can be done separately

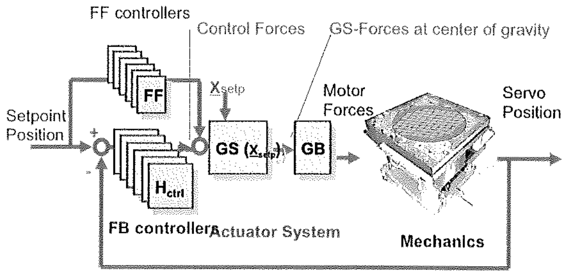
Exercise 4 (20 points)

Consider the wafer steppers built by ASML, i.e.,

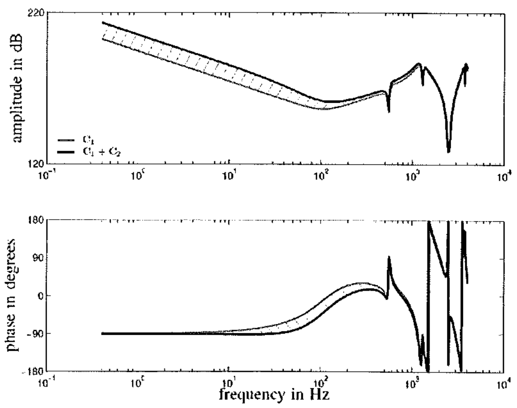




where the right picture shows the long stroke wafer stage, of which the motion needs to be controlled at nanometer level. The general control scheme is given by



a). Designing a PID based feedback controller for a rigid mass system, we obtain the following Bode plot for the real system



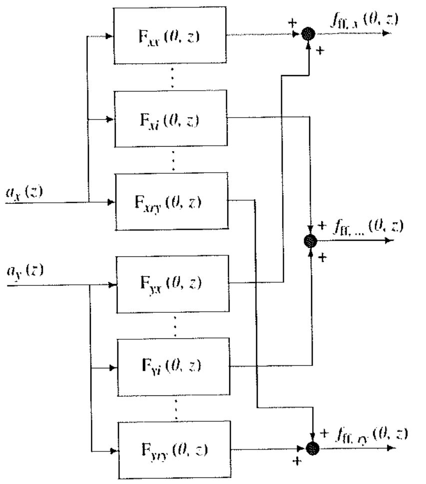
There are resonances at high frequencies. What is the likely reason for these resonances? **(4 points)**

The mass is not rigid, but a bit flexible, hence, resonances at high frequencies may occur

b). What is a mechatronic design solution to deal with resonances in these machines? How should we change/update the design? **(4 points)**

Add sensors and actuators to deal with the resonances (controllability and observability!).

c). Filters are designed for the system with 2 inputs (accelerations), and 12 outputs (forces), i.e.,



Why can't we design Single Input Single Output (SISO) filters for this system? **(4 points)**

There is clear coupling between both inputs and all outputs, hence the transfers influence each other.

d). The precision positioning system at first is able to accelerate, and when it goes to the scanning phase, it needs to be more precise, and thus have less problems with noise. Now consider the following control scheme with a deadzone nonlinearity $\phi(e)$, and the error response, respectively.

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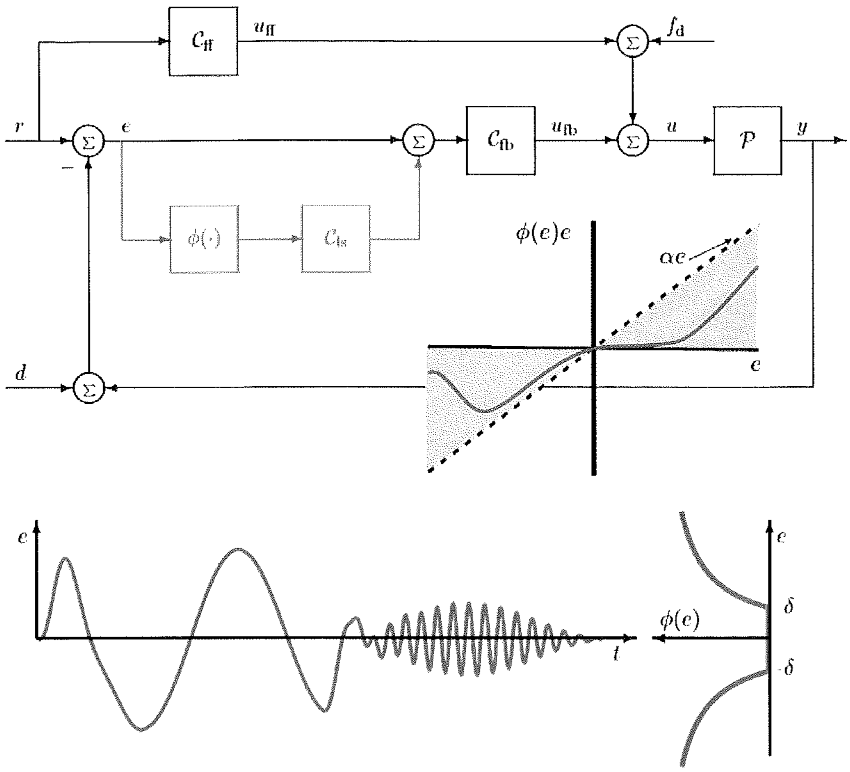
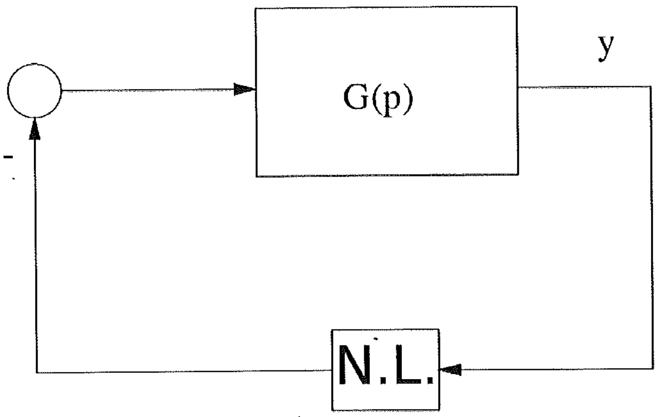
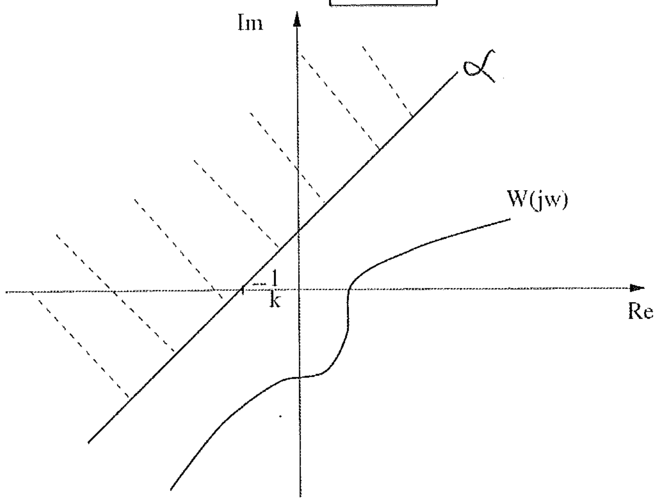


The first part of the error response is the acceleration phase, and the 2nd part the scanning phase. Why is the deadzone nonlinearity useful? **(4 points)**.

The system is able to speed up when ~~a~~ noise is not so harmful, and when scanning starts, it should be more precise. Hence, the deadzone ~~do~~ makes sure that noise is not amplified when not allowed (i.e., in the scanning phase).

e). Now consider the following scheme (left figure) of a linear plant with a nonlinearity (the bottom block) in the loop. Here N.L. is the nonlinear function ϕ. Suppose it is a deadzone nonlinearity.





The right figure is a Popov plot. Please explain the Popov criterium with help of this picture. **(4 points)**.

Popov plot: $W(j\omega) = \cancel{G}\,\underbrace{\mathrm{Re}\{G(j\omega)\}}_{=x} + j\omega\,\underbrace{\mathrm{Im}\{G(j\omega)\}}_{=y}$

consider the line $x - \alpha y + \frac{1}{k} = 0$.

If poles are in the strict LHP, and ϕ belongs to sector $[0,k]$, ~~then~~ if there should exist an α (slope) such that W stays below the line with slope α, then 0 is globally asymptotically stable.

END EXAM